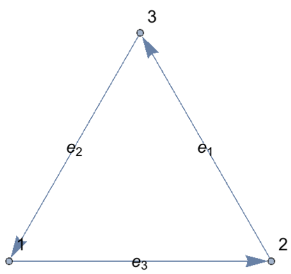
Note: In this problem set, expressions in green cells match corresponding expressions in the text answers.

```
Clear["Global`*"]
```

8 - 13 Find the adjacency matrix of the given graph or digraph.

9.

```
g1 = Graph[{Labeled[1 → 2, "e₃"], Labeled[2 → 3, "e₁"],
   Labeled[3 → 1, "e₂"]}, VertexLabels → "Name", ImageSize → 150]
```

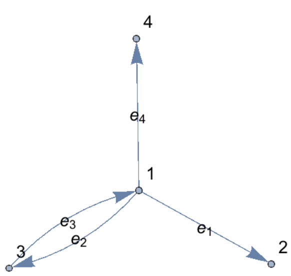



```
ceg = AdjacencyMatrix[g1] // MatrixForm
```

$$\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$$

11.

```
g2 =
 Graph[{Labeled[1 → 2, "e₁"], Labeled[1 → 3, "e₂"], Labeled[3 → 1, "e₃"],
   Labeled[1 → 4, "e₄"]}, VertexLabels → "Name", ImageSize → 150]
```

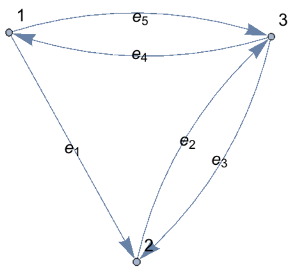



```
ceh = AdjacencyMatrix[g2] // MatrixForm
```

$$\begin{pmatrix} 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

13.

```
g3 = Graph[{Labeled[1 ⟶ 2, "e1"], Labeled[1 ⟶ 3, "e5"],
   Labeled[3 ⟶ 1, "e4"], Labeled[2 ⟶ 3, "e2"], Labeled[3 ⟶ 2, "e3"]},
  VertexLabels → "Name", ImageSize → 150]
```




```
cei = AdjacencyMatrix[g3] // MatrixForm
```

$$\begin{pmatrix} 0 & 1 & 1 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}$$

14 - 15 Sketch the graph for the given adjacency matrix.

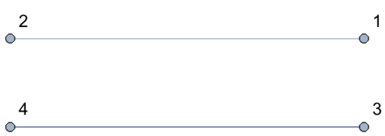
15.

$$\begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

$$\text{cej} = \begin{matrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{matrix}$$

```
{{0, 1, 0, 0}, {1, 0, 0, 0}, {0, 0, 0, 1}, {0, 0, 1, 0}}
```

```
AdjacencyGraph[cej, ImageSize → 200, VertexLabels → "Name"]
```



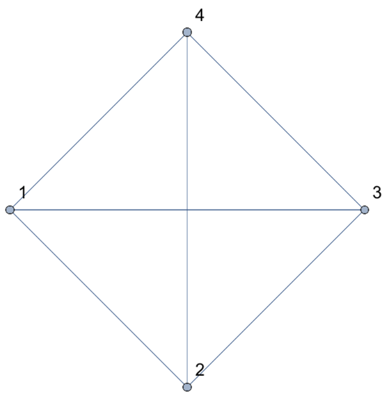
The relationship between vertices in the above sketch is comparable to that of the text, except mirrored.

17. In what case are all the off-diagonal entries of the adjacency matrix of a graph G equal to one?

```
cek = (0 1 1 1
       1 0 1 1
       1 1 0 1
       1 1 1 0)
```

```
{{0, 1, 1, 1}, {1, 0, 1, 1}, {1, 1, 0, 1}, {1, 1, 1, 0}}
```

```
AdjacencyGraph[cek, ImageSize → 200, VertexLabels → "Name"]
```



The example I made suggests that each pair of vertices is joined by an edge, i.e. it is a ***complete*** graph.

19. Incidence matrix $\tilde{B}$ of a digraph. The definition is $\tilde{B} = [b_{jk}]$, where

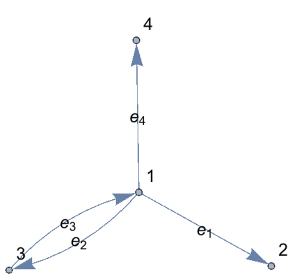
$$\tilde{b}_{jk} = \begin{cases} -1 & \text{if edge } e_k \text{ leaves vertex } j, \\ 1 & \text{if edge } e_k \text{ enters vertex } j, \\ 0 & \text{otherwise} \end{cases}$$

Find the incidence matrix of the digraph in problem 11.

I find that getting the neat table of edges and vertices requires imitating the exact steps shown in the documentation for **IncidenceMatrix**. Specifically, if the **MatrixForm** of g2 is not shielded by a buffer cell from the call for **TableForm**, then **TableForm** will just repeat the **MatrixForm**, as illogical as that seems.

```
Clear["Global`*"]
```

```
g2 =
 Graph[{Labeled[1 ↔ 2, "e1"], Labeled[1 ↔ 3, "e2"], Labeled[3 ↔ 1, "e3"],
   Labeled[1 ↔ 4, "e4"]}, VertexLabels → "Name", ImageSize → 150]
```



```
im1 = IncidenceMatrix[g2] // MatrixForm
```

$$\begin{pmatrix} -1 & -1 & 1 & -1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

```
im1 = IncidenceMatrix[
  g2 = Graph[{Labeled[1 ⧴ 2, "e1"], Labeled[1 ⧴ 3, "e2"],
     Labeled[3 ⧴ 1, "e3"], Labeled[1 ⧴ 4, "e4"]},
    VertexLabels → "Name", ImageSize → 150]]
```

```
SparseArray[ Specified elements: 8
             Dimensions: {4, 4} ]
```

Getting a table which looks a little like the text answer.

```
tek = TableForm[Normal[im1],
  TableHeadings → {VertexList[g2], EdgeList[g2]}]
```

	1 ⧴ 2	1 ⧴ 3	3 ⧴ 1	1 ⧴ 4
1	-1	-1	1	-1
2	1	0	0	0
3	0	1	-1	0
4	0	0	0	1